CHILDREN'S GENERATED WORD PROBLEMS: A CASE STUDY

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This paper investigates the performance of Grade 3 children's generation of word problems from photographs. It looks at the relationship between this performance and performance on a grading instrument; whether children tend to generate characteristic categories of word problems in relation to specific photographs and to what extent student generated word problems reflect their mathematical understandings and thinking.

Ellerton, (1986) wrote that:

> As the substance and style of the problem made up by each child uniquely reflects that child's mathematical experiences and ideas, the made up problem is a particularly useful tool for studying mathematically talented children for whom routine tasks are usually completed quickly and accurately. (p. 261)

Student generated word problems have been investigated by numerous researchers (e.g., Ellerton, 1986; Greenes, Schulman & Spungin, 1992; and Silverman, Winograd & Strohauer, 1992) with children of different ages. Ellerton (1986) had her Grade 7 and 8 students generate and solve difficult problems for their friends. She found that more capable students generated problems of greater complexity than less able students. Providing students with an opportunity to generate mathematics problems achieved a balance between a passive mathematical environment, and one in which an active communication of mathematical ideas became evident. The problems posed also gave an insight into the mathematical understandings of her students. Greenes et al. (1992) required their middle school students (Grades 6 and 7) to generate mathematics stories for the class to solve. Investigated was the relationship between children's understanding of mathematical concepts and their communication of mathematical ideas. The findings suggested that, through discussion, children were able to clarify their concepts and develop an understanding of the mathematics involved in the problems. Silverman et al. (1992), working with Grade 5 students, used student generated word problems to replace reliance upon textbook problems. They initially used word problems from texts, teachers, and students, to introduce problem construction and solution strategies to his class. Group sharing of problems enabled the communication of mathematical ideas and concepts. The complexity of problems generated was not considered in this study.

Fennema and Peterson (1986) have described this complexity. They referred to high-level mathematical tasks as those requiring reflection whilst low-level mathematical tasks required only the use of rote knowledge and the application of an algorithm.

In regard to word problems, the Victorian Curriculum and Standards Framework: Mathematics[1] (Board of Studies, [CSF], 1995) lists several 'learning outcomes[2]' for Level 2 students (Years 3 & 4) namely:

> generate word problems using specified numbers and operations (p. 54);
>
> use mathematical language appropriately ... (p. 102);
>
> generate mathematical questions from ... familiar contexts (p. 104).

The CSF also advances a curriculum focus on generated stories, real situations, pictures and number sentences, that involve collections of objects, money and informal units of measurement. This requires students "to devise situations which may give rise to symbolically expressed operations (e.g., tell a story)." (p. 53) Students are to use their mathematical knowledge and understandings to generate and discuss problems drawn from the mathematics of familiar situations. The question of how this curriculum focus has been applied in the classroom is unknown. The present study seeks to address this lack of knowledge.

STUDY

In this exploratory qualitative research study, the following questions were considered:

1. Can a range of young students generate word problems from photographs?
2. Is there a relationship between a student's ability to generate a word problem and that student's performance on the grading instrument?
3. What is the nature of word problems generated in this way?
4. Do such word problems provide an insight into children's mathematical understandings and thinking?

In this study a word problem has been operationally defined as any problem signifying a mathematical relationship in words to which a solution was to be sought.

METHOD

This study was conducted with a small sample of Year 3 (8-10 year old) children towards the end of their school year. For a week before the study, I visited the class during the mathematics lessons to work as an aide. This allowed the class to get to know me and for me to meet the class and become accustomed to their practice. During this time the teacher gave her students two sheets of commercially produced word problems.

At the beginning of the study, I administered a performance instrument (described in the next section) to the class of 28 children. Each child's performance score was ranked. From the 28 children who completed the instrument, six were selected for the in-depth study (two students from each performance level – above average, average, and below average – see Table 1).

PERFORMANCE INSTRUMENT

This instrument was developed to select children for the interview. It determined their relative performance in solving number problems. The instrument was based upon the arithmetic operations stipulated in the CSF mathematics statement (Board of Studies, 1995). It consisted of 23 number problems. There were 11 high-level and 12 low-level problems. It involved the four arithmetic operations and contained problems that could be derived from the photographs presented to children during the interview. The format of this instrument was based upon that devised by Cai (1995). This format was familiar to students. Previous experience had indicated that:

i. it was typical of class exercises presented by teachers,

ii. the low-level number problems presented in the instrument were typical of those presented to students at this year level,

iii. high-level number problems were not often presented to students at this level.

PHOTOGRAPHS AND TASK

A number of objects (oranges, cutlery, and cupboards) were photographed, scanned and the black and white images electronically enlarged and printed. The prints were laminated. (See Table 3 for the objects on each of the 14 photographs.)

In an individual task setting, the children were told that they would be shown some photographs and asked to "Make up a difficult mathematics word problem or story for your friend to solve". The children were then shown the photographs and asked to select one. They were also told that they need not write down their word problem or story, as I would do that for them. If the child's word problem seemed ambiguous, the child was questioned about its meaning. Once a student had generated a 'story', s/he was invited to provide a solution equation for the problem. The child was then asked to select another photograph. In all, each child selected four photographs, generated word problems and provided solution equations.

RESULTS

Although the six children generated 24 word problems, not all problems were adequate; Two would not be able to be solved by 'friends'. Initially all children had trouble understanding what was required of them: to use the photograph to 'make up a difficult mathematics word problem for your friend to solve'. Once this was understood, all students attempted to generate a word problem. All children showed a lack of command of mathematical language in that they did not have knowledge of mathematical terms other than for the four arithmetic operations (addition, subtraction, multiplication, and division).

Note: As the child ranked 27 was absent from school, it was decided to use the child with the next higher ranking in her place.

Table 1
Students, Ranking, Gender, and Performance Level on the Grading Instrument of the Interview Students.

Student[1]	Student ranking	Gender	Performance level		
			Task Complexity		
			High n=11	Low n=12	Total Score
Mary	1	Female	10	12	22
Noel	2	Male	11	10	21
Olive	18	Female	8	10	18
David	19	Male	8	10	18
Irene	26	Female	5	9	14
Henry	28	Male	5	8	13

Table 1 (above) shows that there were clear differences in the students' performance ranging from near perfect scores to satisfactory but lower scores. The selection of children compared favourably with their teacher's estimate of their achievements. For instance she reported that Irene was not achieving very good results in class, and that Henry had had difficulty meeting the requirements of the Year 3 syllabus. It was fortuitous that the selection of children for the interview consisted of three girls and three boys. With the exception of Noel, the children successfully solved fewer high-level tasks than low-level tasks.

The more capable students (Mary, and Noel) were able to overcome their language difficulties and generate solvable word problems. The average students (Olive, and David) differed in their ability to use language to describe their word problems. Olive showed evident difficulty in verbalising her problems whilst David, apart from the first problem, was able to state his word problems relatively clearly. The two below average students (Irene, and Henry) also differed in their ability to verbalise their word problems. Irene began well but quickly faltered. Henry really found the task difficult and was unable to generate four solvable word problems. All the word problems generated by these six students were representative of low level mathematics activities. They tended to develop word problems in which the answer was the unknown quantity, as opposed to making the unknown either start or change unknown. The word problems and the solution equations generated by the children are presented in Table 2 below.

Although the class teacher had commented that her class "hadn't done a lot of word problems," it was impossible to determine whether this had been the students' only practice with generating word problems. If it were, it would certainly explain their lack of familiarity with the genre and would go some way to explaining the difficulties they experienced with generating a word problem from a photograph.

Table 2
Students, Word Problems, and the Solution Equations Generated.

Student	Word problem
Mary	1. I had one banana and I went to the shop and I bought six more bananas. How many bananas have I got altogether? 1 + 6 = □ 2. I had three people coming over to my house for lunch. I set out a knife, a fork and a spoon for each person. How much cutlery did they get? 3 + 3 + 3 = □ 3. If I had twelve eggs and I broke two, how many eggs would I have left? 12 - 2 = □ 4. If I had four oranges and I ate three oranges and I gave one to my friend, how many would I have left? 4 - 1 = □
Noel	1. What's wrong with one the cupboards in this picture? (No equation required. Draw a picture.) 2. I had six oranges and I found another six on the road. How many oranges do I have altogether? 6 + 6 = □ 3. I had a box of Tri-ominoes. What numbers do you think the numbers go up to in the box? (No equation produced) 4. How much money can you see in the picture below? 20¢ +20¢ + 50¢ + 50¢ + 50¢ + 10¢ + 10¢ + $1-00 = □.
Olive	1. There were four bottles and some one poured some water into five glasses and that bottle was empty. How many bottles were left full of water? 4 - 1 = □ 2. There was a traffic jam because a car smashed into another car and four people were in one car and three people were in another. There were two in the front of each car and one in the back of one car and two in the back of the other and the ambulance came and took the people who were hurt away. How many people do you think got hurt? 7 - 4 = □ 3. Jessy had $12.50 and she bought some Skinny Milk for a dollar. How much change would she have? $12.50 - $1.00 = □ 4. Sam had a banana, apple, orange and sandwich for lunch. How much food altogether? 3 + 1 = □
David	1. There were seven bananas and one was left by itself. How many of the bananas that were left were in the two groups? 7 ÷ 2 = □ 2. There were three spoons, three forks, and three knives. How much cutlery is there altogether? (No equation provided.) 3. There were eight oranges and my friend gave me four more oranges. How many altogether? 8 + 4 = □ 4. I had a sandwich and an orange and my mum gave me another banana and an apple. How much fruit and sandwiches altogether? 2 + 2 = □
Irene	1. .I had twelve oranges and twelve of my friends came and all of my friends liked oranges so they asked if they could have one. And I said, "Yes!" I gave them twelve oranges. How many oranges were left? 12 - 12 = □ 2. It was my birthday party and my mum was baking a cake. On the recipe it said you need to put ten eggs in. My mum put twelve eggs in. How many eggs were left? 12 - 12 = □ 3. Five of my friends came over and it was a really hot day so my mum poured five glasses of water out. One bottle was finished. How many bottles were left? 5 - 5 = □ 4. I went to school and my mum put in my lunch box a sandwich, an apple, an orange and a banana. I ate my sandwich and my orange and my banana but I didn't eat my apple. How many things were left in my lunch box? 4 - 3 = □
Henry	1. I had fourteen oranges and fourteen friends and I squeezed four oranges. Ten oranges were left. How could we make it even for every friend I have? (No equation offered.) 2. It was my birthday party and six of my friends came. How many bananas were left? (No equation offered.) 3. There were twelve juicy oranges and six of them were eaten. How many were left? 12 - 6 = □ 4. There were nine knives, forks and spoons. Six of them needed to be used for dinner. How many were left? 9 - 6 = □

DISCUSSIONS

Generating word problems: Most of the students were able to generate some kind of word problem - albeit with continued probing. Two main areas of difficulty were observed:

1. The children found difficulty in expressing their word problems although it seemed evident from what they were saying that they had an idea of what word problems they wanted to

pose. This could be explained perhaps by a lack of familiarity with the activity. They seemed unable to talk about mathematics.

2. The children generated word problems which were representative of only low-level mathematics activities. If the work-sheets issued to the class by the class teacher a week prior to the project's inception are indicative of the children's exposure to word problems, then there was a reinforcement of both a dependence upon low-level mathematics activities and misconceptions about the four arithmetic operations (Greer, 1992) so typical in word problems often given to students.

Use of photographs: Initially students found the task difficult though once they understood the requirements of the activity they were able to generate a number of word problems. The degree of complexity of the student generated word problems differed. This is evident in the transcripts. Given the seeming lack of experience the students had with word problems in general and the requirement to generate word problems, the task of using a photograph as the basis for generating word problems was not beyond their mathematics capability.

STUDY QUESTIONS

This study, addressed the three research questions. These will now be considered in the light of the study's findings.

1. The success children had with the activity indicated that they could use photographs as the basis for generating a word problem. This success however was limited in that the 'quality' of those word problems generated was not rich in novelty of concepts. Because of a general lack of familiarity with the task, students initially found the task forbidding. However, once an appreciation of the task requirements had been gained and some level of success had been achieved, most students, no matter what their achievement level, were able to generate a word problem representative of the photograph.

2. There seems to be a relationship between the participating children's level of achievement and their generated word problems. All participants experienced some form of mathematics language difficulty but the lower the achievement, the greater the mathematics language difficulty shown. Although they all generated representative low-level word problems – the lowest achiever generated two unsolvable word problems. The solvable problems generated tended to be similar to those provided by the teacher in the work sheets so it is possible that this model of word problems may have inhibited the children's creativity. Most of the word problems were either separating or joining with a few non-routine and unsolvable problems and one of each equal groups and part-part-whole problems.

3. Each of the six students who participated in the interview was able to generate word problems. With respect to Mary and Noel, the above average pair, Mary initially had difficulty understanding what the task required but once she understood Mary was able to generate word problems. Mary generated two joining[4] and two separating word problems. Noel's word problems differed from the other participants' problems in that he generated not only a joining word problem but also two non-routine problems and a part-part-whole word problem. These three required photographs to be seen by his friend; one of these required his friend to discover the difference between the drawers, another required searching through the triangular blocks to discover the highest number, and the third required adding the coins shown in the photograph. The average pair, Olive and David, Olive had difficulty expressing her conception of her word problems but generated three separating and one joining word problems. David, on the other hand seemed to be able to express himself mathematically quite well and generated three joining and one equal groups word problem. In the case of the below average pair, Irene and Henry, Irene, after initial success, found her inadequate mathematics language skills inhibited her verbalisation of the problem concepts she wished to express. The word problems she did generate tended to be inconsistent

with the photographs from which they arose. Henry not only had great difficulty in generating word problems (he generated two unsolvable word problems) but also experienced serious mathematics language difficulty. Of those problems Henry did generate, both were separating problems.

There seems to be a relationship between the participating children's level of achievement and their generated word problems. All participants experienced some form of mathematics language difficulty but the lower the achievement, the greater the mathematics language difficulty shown. Although they all generated representative low-level word problems – the lowest achiever generated two unsolvable word problems. The solvable problems generated tended to be similar to those provided by the teacher in the work sheets so it is possible that this model of word problems may have inhibited the children's creativity. Most of the word problems were either separating or joining with a few non-routine and unsolvable problems and one of each equal groups and part-part-whole problems.

Table 3 below shows the frequency with which the six children selected each photograph. Also listed in the table are the categories of word problems that were generated from each photography. The number in parentheses indicates the number of word problems of each category generated from each photograph.

Table 3
The Frequency of Selection of Each Photograph and the Category of Word Problem Generated from the Photograph

Photograph	**Frequency of selection** (n = 24)	**Category of word problem**
Juice extractor and oranges	1	Unsolvable (1)
Bottles of mineral water	2	Separating (2)
A sandwich and two pieces of fruit	3	Joining (2) Separating (1)
A carton of milk, coins and a docket	2	Part-Part-Whole (1) Separating (1)
Tessellated floor tiles	0	Unrepresented
Kitchen drawers	1	Graphic display (1)
Four grapefruit, one halved.	1	Separating (1)
Seven bananas	3	Joining (1) Partition division (1) Unsolvable (1)
Three knives, three forks and three spoons	3	Joining (2) Separating (1)
Two cartons of eggs	2	Separating (2)
A train track and carriages	1	Separating (1)
A 3 x 4 array of oranges	4	Joining (2) Separating (2)

4. The children's generated word problems tended to mirror those presented immediately before the study commencing. Of the 24 word problems generated, 11 were separating, seven were joining, one of each was part-part-whole, non-routine, graphic, or partition division. Two problems were unsolvable. Not one of the solvable problems involved high-level mathematical tasks.

The unfamiliarity of the task appeared to hamper an adequate reflection of the children's understandings and thinking to be revealed. As stated above all solvable problems were low-level mathematical activities showing no evidence of metacognitive thinking. One child (Noel)

generated two problems requiring drawings to be viewed to answer his questions. This question raises some implications for mathematics education within schools.

Posing word problems appeared to be a novel experience for these children. Whilst all their problems were low-level word problems, the children who experienced the greatest difficulty completing the task were those of below average achievement. There was, however, no evidence to suggest that the word problems posed by the above average children were of any greater complexity than those posed by the average or below average pairs. This differed from Ellerton's (1986) findings. This could be explained by the novelty of the situation and that these children did not have an opportunity to communicate their problems to their cohort and so gain valuable feedback. Because of this lack of communication, so valuable to Greenes et al. (1992) and Silverman et al. (1992) the children were unable to clarify their understanding of the mathematics involved.

FURTHER RESEARCH

Although this study involved only a small sample of children from one class, teaching experience suggests that the findings would be similar across a number of classes and schools. Further research to determine whether this is the case is obviously needed. It would also be useful to investigate, for example,

i. whether two children working collaboratively could generate higher level problems;

ii. if several experiences of problem-posing would improve children's problem posing;

iii. whether exposure to stimuli other than photographs would make a difference;

iv. the kind and degree of support teachers might need to incorporate problem-posing into their mathematics' syllabus.

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Endnotes

1 The CSF provides schools with formal guidelines for curriculum development in eight Key Learning Areas.

2 This term is used in the CSF to indicate what students are expected to have achieved at the completion of each level.

3 Pseudonyms have been used.

4 For a full description of word problem categories see Carpenter & Moser (1982) and Ellerton (1986).